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MULTIPLICITY-FREE PERMUTATION CHARACTERS OF COVERING GROUPS OF SPORADIC SIMPLE GROUPS

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ABSTRACT. In this paper we classify all multiplicity-free faithful permutation representations of the covering groups of the sporadic simple groups. These results were obtained computationally, making extensive use of the **GAP** library of character tables.

1. INTRODUCTION

Any permutation representation of a group G can be extended to a linear representation (on a space with basis in bijection with the set permuted) over any field, in particular over the complex numbers $\mathbb{C}$. One can then consider the decomposition of this linear representation into its irreducible constituents. The structure of this decomposition is linked to the structure of the permutation representation. For instance the multiplicity of the trivial character gives the number of orbits, and, for transitive representations, the sum of the squares of the multiplicities of the constituents gives the rank (the number of orbits of the point stabilizer).

Of particular interest are those cases where the multiplicities of all the constituents are one, called *multiplicity-free* representations. This condition is related to two natural permutation-theoretic conditions as follows:

$$\text{generously transitive} \implies \text{multiplicity-free} \implies \text{transitive}$$

(cf. [2] p63). Multiplicity-free permutation representations can also be characterized as those whose centralizer algebra is abelian [5]. Also, the action of the automorphism group on the vertices of any distance transitive graph is always multiplicity-free, so that a classification of multiplicity-free permutation representations serves as an initial step towards a classification of distance-transitive graphs with a given automorphism group. General results on the classification of distance-transitive graphs [6] enable many cases to be reduced to graphs whose automorphism groups are simple or *almost simple* (simple modulo a cyclic central subgroup and/or some extending automorphisms).

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The classification of Finite Simple Groups arranges all simple groups into four *families*: cyclic groups of prime order; alternating groups; finite groups of Lie type and the 26 sporadic groups, and it is this last family which is our focus here. The multiplicity-free permutation representations of the sporadic groups and their automorphism groups are completely known [5, 1] and tabulated in [1].

This paper is intended as a sequel to [1]. We give the multiplicity-free, faithful permutation characters of the covering groups $m{\cdot}G$ of the sporadic simple groups, where G is a sporadic simple group and m divides the order of its Schur multiplier (given, for instance, in the Atlas of Finite Groups [3]). The following sporadic groups have trivial Schur multipliers and so do not possess nontrivial covering groups: $M_{11}, J_1, M_{23}, M_{24}, He, Co_3, Co_2, HN$, Ly, Th, Fi_{23}, J_4 and M.

In Section 2, we give some background and prove some results which have been used to obtain the desired permutation characters. We also deal with the computational techniques that have been used in determining the multiplicity-free and faithful representations. In Section 3, we present the results in tables. In Section 4, we give a more detailed explanation of certain complex cases and prove the one result for which our general methods were inadequate.

2. Preliminaries

Proposition 2.1. *Let $\overline{G}$ be a group of shape $m{\cdot}G$ and $\phi : \overline{G} \longrightarrow G$ the natural homomorphism from $\overline{G}$ to $G \cong \overline{G}/Z$, where Z is the centre of $\overline{G}$. If $\overline{G}$ has a multiplicity-free faithful permutation representation with point stabilizer $\overline{H}$, where $H = \overline{H}\phi$ then the permutation action of G on the cosets of H is also multiplicity-free and faithful.*

Proof. The action of G on the cosets of H is equivalent to that of $\overline{G}$ on the cosets of $\overline{H}Z$ (on which Z will act trivially). The bijection is provided by $\psi : \overline{H}Zx \longrightarrow Hx\phi$, which is easily checked to be well-defined, surjective and injective. This, with ϕ is an equivalence of permutation representations, since

$$\overline{H}Zxy\psi = H(xy)\phi = Hx\phi y\phi = \overline{H}Zx\psi y\phi.$$

Now in the space V with basis defined on the cosets of $\overline{H}$ in $\overline{G}$, consider the subspace W spanned by the vectors

$$w_x = \sum_{z \in Z} \overline{H}xz$$

where $x \in \overline{G}$. Two such vectors w_x and w_y are equal if and only if x and y lie in the same coset of $\overline{H}Z$, and since Z is central in $\overline{G}$ it is easy to check that W is invariant under $\overline{G}$, furthermore, the action of $\overline{G}$ on W is just that on the space spanned by the cosets of $\overline{H}Z$.

Thus the permutation representation of G on H appears as a submodule of that of $\overline{G}$ on $\overline{H}$ and so, if the latter is multiplicity-free and faithful, then the former certainly must be. □

Proposition 2.1 above allows us to take as the starting point for our classification, the classification in [1] of multiplicity-free permutation representations for the sporadic groups, and consider in each case its possible extensions.

We now consider a candidate permutation representation of a simple group G with point stabilizer H and consider how it might extend to a faithful representation of $\overline{G}$. This is determined by the subgroup $K = H\phi^{-1}$ of $\overline{G}$. If the permutation representation extends, then the point stabilizer $\overline{H} \leq \overline{G}$ will certainly be a subgroup of K. Furthermore, we must have $\overline{H}\phi = H$ and $\overline{H} \cap Z = \{1\}$. In fact there will be one equivalence class of extension for each $\overline{G}-$conjugacy class of such subgroup in K. Note that, in this situation ϕ actually provides an isomorphism between $\overline{H}$ and H.

Remark 2.2. *If Z has a proper subgroup Y, then $\overline{G}/Y$ will also be a covering group of G and it is easy to check that a (multiplicity-free) permutation representation of G can only extend to a faithful (multiplicity-free) permutation representation of $\overline{G}$ if it also extends to a faithful (multiplicity-free) permutation representation of $\overline{G}/Y$.*

In all the cases we will consider, the central subgroup Z of $\overline{G}$ is cyclic of order m, say. Let z be a generator of Z. We can obtain useful further information about the decomposition of a permutation representation by considering the action of z. Let V be the complex vector space on which $\overline{G}$ acts by permuting the basis. Then V must decompose into eigenspaces: $E_0, E_1, \ldots, E_{m-1}$ where z acts as η^i on E_i (η a primitive mth root of unity).

We can make various observations about this decomposition.

- In a transitive faithful permutation action, no nonidentity element of Z can fix any points since
$$\alpha z = \alpha \Longrightarrow \alpha g z = \alpha z g = \alpha g$$
which, with transitivity, would imply that z fixes every point, contradicting faithfulness. Thus z acts as a product of m-cycles.

- Let $(\alpha_0, \alpha_1, \ldots, \alpha_{m-1})$ be a cycle of z. Then it is easy to see that
$$\sum_{i=0}^{m-1} \alpha_i \eta^{ij} \in E_j.$$

From this, we can see that $dimE_i = dimV/m$.

- Since Z is central, each E_i is invariant under $\overline{G}$.

- The action of $\overline{G}$ on E_i has kernel generated by $z^{(i,m)}$ and can contain no irreducible constituent which is not faithful modulo this kernel.

- Finally, complex conjugation obviously exchanges E_i and E_{-i} (indices taken modulo m). Since the action of $\overline{G}$ on V is given by real (permutation) matrices, however, complex conjugation must preserve the $\overline{G}$-module structure of the E_i, so whatever irreducible constituents make up E_i, their complex conjugates must make up E_{-i}.

- The subspace W considered above is simply E_0.

In the particular cases we are considering, these observations have the following consequences:

- when $m = 2$, V is the direct sum of two components of equal dimension, one composed entirely of characters of G, the other of faithful characters of $2{\cdot}G$

- when $m = 3$, V is the direct sum of three components of equal dimension, one composed entirely of characters of G, the other two complex conjugates of one another and composed entirely of faithful characters of $3{\cdot}G$

- when $m = 4$, V is the direct sum of four components of equal dimension, one composed entirely of characters of G, one composed of faithful characters of $2{\cdot}G$ and the other two complex conjugates of one another and composed entirely of faithful characters of $4{\cdot}G$

- when $m = 6$, V is the direct sum of six components of equal dimension, one composed entirely of characters of G, one composed of faithful characters of $2{\cdot}G$ and the others making up two pairs of complex conjugates, one composed of faithful characters of $3{\cdot}G$ and the other of $6{\cdot}G$

- when $m = 12$, V is the direct sum of twelve components of equal dimension, one composed entirely of characters of G, one composed of faithful characters of $2{\cdot}G$ and the others making up five pairs of complex conjugates, one composed of faithful characters of $3{\cdot}G$, one of $4{\cdot}G$, one of $6{\cdot}G$ and the other two of $12{\cdot}G$

Remark 2.2 above implies that when m is composite, we need only consider extensions of representations that have already extended to $k{\cdot}G$ when k divides m.

Our main techniques for obtaining these results are character-theoretic, and make extensive use of the character table library of the **GAP** [4] system, which in particular includes the character tables of all the sporadic simple groups, their covering groups and many of their maximal subgroups.

In the smaller cases, we make use of the **PermChars** function of **GAP** to list all combinations of irreducible characters which have the correct degree, and pass a number of tests for being permutation characters. In [1], all characters satisfying the properties of [1, Lemma 4.3] are called possible permutation characters. We have to be aware of two pitfalls here: firstly, the tests are not complete and some characters pass them all, but are not characters of any permutation representations; secondly, inequivalent permutation representations may give rise to the same character. Nevertheless, if there are no multiplicity-free candidate permutation characters, then we know that there is no multiplicity-free permutation representation and if there is just one candidate permutation character, and we can exhibit a subgroup of that index, then we know that the corresponding action must afford that character. Furthermore faithfulness of a representation of a covering group can be checked both in the character (which must vanish on nonidentity elements of the centre) and from the subgroup, which must meet the centre only trivially. Thus, if we exhibit a faithful representation, and have just one faithful character, we again know that they must correspond.

Multiplicity-free permutation characters can be determined using the tables of marks. [1, Appendix 1] gives a **GAP** procedure for computing multiplicity-free permutation characters using the tables of marks.

In the larger cases, we use character induction, since in all but one case, the character table of the appropriate subgroups are available (see Section 4 for the details of this one case). Here we use the **PossibleClassFusions** function of **GAP** to find all possible embeddings of H into $\overline{G}$ and then compute, for each one, the character $1_H \uparrow^{\overline{G}}$. One of these would have to be the character of any permutation representation with point stabilizer isomorphic to H. In practice, sufficiently few possible class fusions were found in each case, that there was no problem identifying the correct permutation character(s).

[1, Lemma 4.4] describes a technique for determining all multiplicity-free characters $\chi \uparrow^G$ for χ a possible permutation character of a maximal subgroup H of a group G. The corresponding **GAP** procedure for this technique is given in [1, Appendix 2].

3. Results

We present the results in tables with various columns. The first column describes the isomorphism type of the point stabilizer subgroup, which, as noted above, must be the isomorphism type of the corresponding point stabilizer in the simple group.

The second column refers to the index in $m{\cdot}G$ of the group in column 1. In tables for $2{\cdot}G$ and $3{\cdot}G$, the third column refers to the line number in the relevant table in [1] which describes the corresponding permutation representation of the simple group G. In tables for $6{\cdot}G$, the third and fourth columns give the row numbers of the corresponding representations in the tables for $2{\cdot}G$ and $3{\cdot}G$ respectively. The next column gives the decomposition of the faithful part of the permutation module. The decompositions of the non-faithful parts are found in the indicated rows of the tables for G (in [1]), $2{\cdot}G$ and $3{\cdot}G$. In giving these decompositions, we denote by e.g $123a$ the first *faithful* representation of degree 123 in the Atlas [3]. In the case of $3{\cdot}G$ or $6{\cdot}G$ where each row of the Atlas table corresponds to two complex conjugate characters (indicated there by $\circ 2$), we call the first e.g $123b$ and the other $123b'$. The last column gives the rank of the permutation action. Other notations follow the same format and style as used in [1].

All computations were performed using **GAP 4** running on a Pentium III computer at the Centre for Interdisciplinary Research in Computational Algebra at the University of St Andrews.

The following covers of sporadic simple groups have no multiplicity-free permutation representations and therefore no corresponding tables below: $3{\cdot}J_3, 4{\cdot}M_{22},\ 12{\cdot}M_{22}$

$2{\cdot}M_{12}$

M_{11}	24	1	$+12a$	3
M_{11}	24	2	$+12a$	3
$A_6{\cdot}2_1$	264	5	$+12a+120a$	7
$A_6{\cdot}2_1$	264	8	$+12a+120a$	7
$3^2{\cdot}2{\cdot}S_4$	440	11	$+110ab$	7
$3^2{\cdot}2{\cdot}S_4$	440	13	$+110ab$	7
$3^2{\cdot}2{\cdot}S_4$	440	11	$+12a+44ab+120a$	9
$3^2{\cdot}2{\cdot}S_4$	440	13	$+12a+44ab+120a$	9
$3^2{\cdot}2{\cdot}A_4$	880	12	$+12a+44ab+110ab+120a$	14
$3^2{\cdot}2{\cdot}A_4$	880	14	$+12a+44ab+110ab+120a$	14

$2{\cdot}M_{22}$

$2^4{:}A_5$	924	3	$+126ab + 210a$	8
A_7	352	4	$+56a + 120a$	5
A_7	352	5	$+56a + 120a$	5
$2^3{:}L_3(2)$	660	7	$+120a + 210a$	7

$3{\cdot}M_{22}$

$2^4{:}A_5$	1386	3	$+21aa' + 105aa'bb' + 231aa'$	13
$2^4{:}S_5$	693	6	$+21aa' + 105aa'bb'$	10
$2^3{:}L_3(2)$	990	7	$+21aa' + 99aa' + 105aa'bb'$	13
$L_2(11)$	2016	9	$+21aa' + 105aa'bb' + 210aa' + 231aa'$	16

$6{\cdot}M_{22}$

$2^4{:}A_5$	2772	1	1	$+126aa'bb' + 210aa'$	22
$2^3{:}L_3(2)$	1980	4	3	$+330aa'$	17

$2{\cdot}J_2$

$U_3(3)$	200	1	$+50ab$	5

$2{\cdot}HS$

$U_3(5)$	704	3	$+176ab$	6
$U_3(5)$	704	5	$+176ab$	6
A_8	4400	8	$+176ab + 924ab$	13
M_{11}	11200	10	$+56a + 176ab + 616ab + 1980ab$	16
M_{11}	11200	11	$+56a + 176ab + 616ab + 1980ab$	16

$3{\cdot}McL$

$2{\cdot}A_8$	66825	6	$+2772aa' + 5103aa' + 6336aa' + 8064aa'$	14

$2{\cdot}Ru$

${}^2F_4(2)'$	16240	2	$+28ab + 4032ab$	9

$2{\cdot}Suz$

$U_5(2)$	65520	4	$+364ab + 16016ab$	10

$3{\cdot}Suz$

$G_2(4)$	5346	1	$+66aa' + 1716aa'$	7
$U_5(2)$	98280	4	$+78aa' + 1365aa' + 4290aa' + 27027aa'$	14
$2_-^{1+6}{\cdot}U_4(2)$	405405	5	$+66aa' + 429aa' + 1716aa' + 6720aa'$	23
			$+18954aa' + 42900aa' + 64350aa'$	
$2^{4+6}{:}3A_6$	1216215	6	$+1365aa' + 4290aa' + 27027aa' + 42900aa'$	27
			$+85800aa' + 104247aa' + 139776aa'$	

$6{\cdot}Suz$

$U_5(2)$	196560	1	2	$+12aa' + 924aa' + 4368aa' + 27456aa'$	26

$3{\cdot}O'N$

$L_3(7){:}2$	368280	1	$+495aa' + 58653aa' + 63612aa'$	11
$L_3(7){:}2$	368280	3	$+495bb' + 58653aa' + 63612aa'$	11
$L_3(7)$	736560	2	$+495aa' + 58653aa' + 63612aa' + 122760aa'$	15
$L_3(7)$	736560	4	$+495bb' + 58653aa' + 63612aa' + 122760aa'$	15

$2{\cdot}Fi_{22}$

$O_7(3)$	28160	2	$+352a + 13728a$	5
$O_7(3)$	28160	3	$+352a + 13728b$	5
$O_8^+(2){:}S_3$	123552	4	$+13728a + 48048a$	6
$O_8^+(2){:}S_3$	123552	4	$+13728b + 48048b$	6
$O_8^+(2){\cdot}3$	247104	5	$+13728ab + 48048ab$	11
$O_8^+(2){\cdot}2$	370656	6	$+352a + 13728a + 48048a + 123200a$	11
$O_8^+(2){\cdot}2$	370656	6	$+352a + 13728b + 48048b + 123200a$	11

$3{\cdot}Fi_{22}$

$O_8^+(2){:}S_3$	185328	4	$+351aa' + 19305aa' + 42120aa'$	10
$O_8^+(2){\cdot}3$	370656	5	$+27027aa' + 96525aa'$	11
$O_8^+(2){\cdot}3$	370656	5	$+351aa' + 7722aa' + 19305aa'$	17
			$+42120aa' + 54054aa'$	
$O_8^+(2){\cdot}2$	555984	6	$+351aa' + 19305aa' + 27027aa'$	17
			$+42120aa' + 96525aa'$	
$2^6{:}S_6(2)$	2084940	8	$+351aa' + 19305aa' + 27027aa' + 42120aa'$	24
			$+96525aa' + 123552aa' + 386100aa'$	
${}^2F_4(2)'$	10777536	9	$+19305aa' + 27027aa' + 51975aa'$	25
			$+386100aa' + 405405aa' + 1351350cc'dd'$	

$6{\cdot}Fi_{22}$

$O_8^+(2){:}S_3$	370656	3	1	$+61776aa'$	14
$O_8^+(2){:}S_3$	370656	4	1	$+61776bb'$	14
$O_8^+(2){\cdot}3$	741312	5	3	$+61776aa'bb'$	25
$O_8^+(2){\cdot}2$	1111968	6	4	$+61776aa' + 123552aa'$	25
$O_8^+(2){\cdot}2$	1111968	7	4	$+61776bb' + 123552aa'$	25

$2{\cdot}Co_1$

Co_2	196560	1	$+24a + 2576a + 95680a$	7
Co_3	16773120	5	$+24a + 2576a + 95680a + 1841840a$ $+6446440a$	12

$3{\cdot}Fi_{24}'$

Fi_{23}	920808	1	$+783aa' + 306153aa'$	7
$O_{10}^-(2)$	150532080426	2	$+783aa' + 64584aa' + 306153aa'$ $+6724809aa' + 19034730aa' + 43779879aa'$ $+195019461aa' + 203843871aa'$ $+1050717096aa' + 1818548820aa'$ $+10726070355aa' + 15016498497aa'$ $+21096751104aa'$	43

$2{\cdot}B$

Fi_{23}	2031941058560000	4	$+96256a + 10506240a + 410132480a$ $+8844386304a + 36657653760a$ $+864538761216a + 4322693806080a$ $+10177847623680a + 60780833777664a$ $+110949141022720a + 828829551513600a$	34

Except for the two largest ones ($3{\cdot}Fi_{24}'$ on $O_{10}^-(2)$ and $2{\cdot}B$ on Fi_{23}), explicit permutations have been constructed for all of these representations (some were already known) and are available from the first author.

4. Remarks on specific cases

There are two cases in the above tables where the results reflect especially complex situations in the groups concerned, and one where a special proof technique was needed.

According to [1], M_{12} has two classes of maximal subgroups isomorphic to $A_6{\cdot}2^2$, two classes of non-maximal subgroups isomorphic to $A_6{\cdot}2_1 \cong S_6$ and two classes isomorphic to $A_6{\cdot}2_2 \cong PGU_2(9)$. In each case, the two classes are exchanged by the outer automorphism of M_{12} and each class affords a multiplicity-free permutation character. It is easy to see that M_{12} must also have two classes of subgroups isomorphic to $A_6{\cdot}2_3 \cong M_{10}$, but these do not afford a multiplicity-free permutation character. Again examining [1] we see that the two conjugacy classes of subgroups isomorphic to $A_6{\cdot}2_1 \cong S_6$ actually afford the same permutation character (although inequivalent permutation representations).

The **GAP** function **PermChars** actually reveals five distinct multiplicity-free possible permutation characters of degree 132 for M_{12}, and we conclude that two of them cannot actually be given by any permutation representations.

In $2{\cdot}M_{12}$, we find that the preimage of $A_6{\cdot}2^2$ is a group H which can be described as $(2 \times A_6){\cdot}2^2$. The largest solvable quotient of this: $H/H^{(\infty)}$, is isomorphic to D_8. Within this, we find that the preimage of a subgroup $A_6{\cdot}2_1$ is a subgroup H_1 of shape $2 \times A_6{\cdot}2_1$ while that of a subgroup of shape $A_6{\cdot}2_2$ is H_2 of shape $(2 \times A_6){\cdot}2$ with the outer part affording the 2_2 automorphism and squaring to the centre.

Now H has no subgroup of index 2 not containing the centre (as may be seen in D_8) and so there are no faithful extensions of the permutation action of M_{12} on 66 points. H_1 has two conjugacy classes of subgroups of index 2, but they are conjugate in H, so we see just one equivalence class of permutation representations (and its image under the outer automorphism), which turn out to be multiplicity-free. H_2 again has no subgroup of index 2 not containing the centre.

Thus we see in our table just two equivalence classes of faithful multiplicity-free permutation representations of $2{\cdot}M_{12}$ with point stabilizers isomorphic to $A_6{\cdot}2_1$, exchanged by the outer automorphism of $2{\cdot}M_{12}$ and affording the same character.

In Fi_{22}, there is a conjugacy class of subgroups isomorphic to $O_8^+(2){:}S_3$ which extends to $2 \times O_8^+(2){:}S_3$ in $Fi_{22}{\cdot}2$. The permutation representation of Fi_{22} on the cosets of $O_8^+(2){:}S_3$ is multiplicity-free.

In $2{\cdot}Fi_{22}$, the preimage of a subgroup in this class is isomorphic to $2 \times O_8^+(2){:}S_3$, and so has two non-conjugate subgroups of index 2 not containing the central element, leading to two multiplicity-free faithful permutation representations of $2{\cdot}Fi_{22}$. However, it can be seen from the character decompositions that these are exchanged by the outer automorphism.

Thus, the preimage of $2 \times O_8^+(2){:}S_3$ in (either isomorphism type of) $2{\cdot}Fi_{22}{\cdot}2$ must be a group $(2 \times O_8^+(2){:}S_3){\cdot}2$ in which the outer involution acting by conjugation "shifts" elements in the outer half of $2 \times O_8^+(2){:}S_3$ by the central element of that group.

Exactly the same phenomenon occurs for the non-maximal subgroup $O_8^+(2){\cdot}2$.

The non-maximal subgroup $O_8^+(2){\cdot}3$ has preimage isomorphic to $3 \times O_8^+(2){\cdot}3$ in $3{\cdot}Fi_{22}$. This group has two conjugacy classes of subgroups of index 3 not containing the centre, which give rise to inequivalent faithful multiplicity-free permutation representations of $3{\cdot}Fi_{22}$, one of rank 11 and the other rank 17.

Finally, the character table of the maximal subgroup $2_+^{1+22}{\cdot}Co_2$ of B is not known, so we could not use our normal character induction methods. Instead, we look at the degrees of the faithful characters of $2{\cdot}B$ and can easily check that no combination of these (without repetition) can sum to 11707448673375. Thus there can be no faithful multiplicity-free permutation representation of $2{\cdot}B$ with this stabilizer.

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